Hyperon Nonleptonic Weak Decays Revisited

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We first review the current algebra - PCAC approach to nonleptonic octet baryon 14 weak decay B $\rightarrow B'\pi$ amplitudes. The needed four parameters are independently determined by $\Omega \rightarrow \Xi\pi, \Lambda K$ and $\Xi^- \rightarrow \Sigma^-\gamma$ weak decays in dispersion theory tree order. We also summarize the recent chiral perturbation theory (ChPT) version of the eight independent B $\rightarrow B'\pi$ weak $\Delta I = 1/2$ amplitudes containing considerably more than eight low-energy weak constants in one-loop order.

I. INTRODUCTION

For about thirty-five years standard current algebra - partially conserved axial current (CA-PCAC) studies have been used to describe hyperon nonleptonic weak decays. Specifically the measured [1] seven s-wave B $\rightarrow B'\pi$ amplitudes were roughly explained [2] using CA-PCAC and SU(3) symmetry for a $(d/f)_w$ ratio of ∼-0.4. Then the observed [1] p-wave B $\rightarrow B'\pi$ amplitudes were predicted using a baryon pole model with pseudoscalar couplings [3] for a different $(d/f)_w$ ratio of ~ -0.8. This s-wave p-wave mismatch was roughly eliminated when decuplet intermediate state pole graphs were included [4].

Related techniques were used to calculate weak strangeness violating vertices, such as NNK, which were needed in the description of hypernuclear decays. Thus it might be useful to revisit this particular approach.

More recently, an alternative chiral approach has been used to study B $\rightarrow B'\pi$ weak decays, with pseudoscalar - baryon PBB couplings replaced by pseudovector PBB couplings in a chiral perturbation theory (ChPT) heavy baryon formulation context [5, 6]. However the s-wave, p-wave amplitude mismatch is still a problem for ChPT [5-7]. While decuplet *loops* are folded into the analysis, only the usual low energy constants (LECs) are considered in leading (non-analytic) log approximation in refs. [5, 6], but additional counterterms are included in ref. [7]. In fact [7] accounts for *all* terms at one loop order, which includes approximately 20 counterterm terms in their eq. (41), 34 momentum-dependent divergence terms in eq. (44), 18 mesonic lagrangian terms in eq. (B.6), 17 second-order lagrangian terms in (B.8), 38 second-order lagrangian terms in (B.9), 88 relativistic correction terms in (B.10), 33 momentum-dependent divergence terms in (B.17), 27 Z-factor terms in (C.1), 55 terms in the p-wave $\Sigma^+ n$ amplitude in (D.1), 40 terms in the p-wave $\Sigma^- n$ amplitude (D.2), 35 terms in the p-wave Λp amplitude (D.3) and 37 terms in the p-wave amplitude (D.4).

In this paper we return to the original CA-PCAC picture of B $\rightarrow B'\pi$ weak decays as formulated in [2, 3] and then resolved in refs. 4. First in Sec. 2 we review the chiral, $\Delta I = 1/2$ and SU(3) octet structure of B $\rightarrow B'\pi$ weak amplitudes. Next in Sec. 3 we fold in the decuplet (D) resonance corrections in a dispersion theory context. We also confirm the $< B|H_w|D >$ weak scales, h_2, h_3. Finally in Sec. 4 we resolve the long-standing mismatch between the s-wave and p-wave B $\rightarrow B'\pi$ weak amplitudes. We give our conclusions in Sec. 5. In Appendix A we list the various SU(3) octet and decuplet pole amplitudes needed for this CA-PCAC analysis. In Appendix B we list instead the various resonance contributions to nucleon pion-photoproduction; the latter demonstrating that the Δ decuplet resonance plays a dominant role, far more important than $\frac{1}{2}^-$ resonances. The same pattern should also hold for hyperon nonleptonic weak decays.

II. CHIRAL, SU(3) AND ΔI = 1/2 STRUCTURE

The CA-PCAC approach employs the chiral commutator $[Q + Q_5, H_w] = 0$, for a weak hamiltonian density built up from V-A currents. Then the resulting charge commutator amplitude M_{cc} for $f_\pi \approx 93$ MeV satisfies:

$$if_\pi M_{cc} = - < B^f|[Q_5^j, H_w]|B^i >= if^{fjb} < B^b|H_w|B^i > -if^{jia} < B^f|H_w|B^a >, \tag{1}$$

where the parity-violating (pv) and parity-conserving (pc) amplitudes (between baryon spinors) are [8]

$$M =< B'\pi|H_w^{pv} + H_w^{pc}|B >= iA + \gamma_5 B. \tag{2}$$

With H_w^{pc} transforming like λ_6 we invoke [9] $< B'|H_w^{pv}|B >= 0$, along with the SU(3) structure

$$< B^f|H_w^{pc}|B^i >= h_1(d_w d^{f6i} + f_w i f^{f6i}), \tag{3}$$

for the scale $h_1 \approx 22eV$ *predicted* in Sec. 4 coupled with $(d/f)_w \approx -0.811$ and $d_w + f_w = 1$, the former near the fitted octet pc pole amplitude of refs. [3,4,10]. We list the various SU(3) scales from (3) in Appendix A.

Next to separate off the rapidly varying pole amplitudes $M = M^P + \overline{M}$ we follow the soft-pion method (pseudoscalar PBB coupling) and determine the (slowing varying) background $\overline{M}$:

$$M = M(0) + M^P - M^P(0), \tag{4}$$

where $M(0) = M_{cc}$ from (1) and $M^P(0)$ is evaluated at the soft point $p_\pi = 0$. Then the seven pv s-wave amplitudes have the leading form found from CA-PCAC using (1) and (2):

$$A_{cc} = -\frac{1}{f_\pi}[if^{fjb} < B^b|H_w^{pc}|B^i > -if^{jia} < B^f|H_w^{pc}|B^a >]. \tag{5}$$

Also the seven pc p-wave amplitudes are dominated by the rapidly varying octet pole terms [4]:

$$B_8 = -(m_f + m_i)\sum_n \left(\frac{g^{fn}H_{pc}^{ni}}{(m_i - m_n)(m_f + m_n)} - \frac{H_{pc}^{fn}g^{ni}}{(m_n - m_f)(m_i + m_n)}\right). \tag{6}$$

In equation (6) we employ the strong interaction PBB pseudoscalar coupling constants $g^{\pi fi}$ listed in Table I with d/f ratio [4] $(d/f)_P \approx 2.1$ and $(d+f)_P = 1$ so that $d_P = 0.678$, $f_P = 0.322$.

Table I: Strong coupling constants

$g_{\pi^- pn}, g_{\pi^+ np}$	$= \sqrt{2}g \approx 19.0$
$g_{\pi^o pp}$, $-g_{\pi^o nn}$	$= g \approx 13.4$
$g_{\pi\Sigma\Lambda}$	$=\frac{2}{\sqrt{3}}d_p g \approx 10.5$
$g_{\pi^o\Sigma^+\Sigma^+}$, $g_{\pi^-\Sigma^o\Sigma^-}$, $-g_{\pi^o\Sigma^o\Sigma^+}$	$= 2f_p g \approx 8.6$
$g_{\pi^o\Xi^o\Xi^o}$	$= g(f-d)_p \approx -4.8$
$g_{\pi^-\Xi^o\Xi^-}$	$\sqrt{2}g(d-f)_p \approx 6.7$

However decuplet intermediate states contribute at the 30% level and we consider A_{10} and B_{10} amplitudes and couplings in Sec. 3.

Lastly we should fold in $\Delta I = 1/2$ and SU(3) breaking corrections. The present s and p wave $B \to B'\pi$ data [1] listed in Table II shows that the $\Delta I = 1/2$ rules $\Lambda^o_- = -\sqrt{2}\Lambda^o_o$ and $\Xi^-_- = -\sqrt{2}\Xi^o_o$ are well satisfied both for s and p waves. However $\Sigma^+_o = (\Sigma^+_+ - \Sigma^-_-)/\sqrt{2}$ is violated at the 10% (20%) level for s waves (p waves), as is the Lee-Sugawara [11] SU(3) sum rule $\Sigma^+_o = (\Lambda^o_- + 2\Xi^-_-)/\sqrt{3}$. For simplicity we ignore $\Delta I = 1/2$ corrections (transforming like *27*, the group theory language used in refs [4]) but fold in larger SU(3)-breaking $\Lambda(1405)$ terms contributing only to Σ^+_+ and Σ^-_- amplitudes. Thus the final forms for our s and p wave amplitudes will be

$$A = A_{cc} + A_{10} + A_{\Lambda'}, B = B_8 + B_{10} + B_{\Lambda'}. \tag{7}$$

At the quark level, the dominant W-exchange and smaller W self-energy single quark line diagrams [12] naturally transform as $\Delta I = 1/2$, as first shown by Koerner and Pati and Woo [13]. Nevertheless our predicted hadron level amplitudes displayed in Appendix A will have the form of eqs. (7) expressed in terms of the strong coupling constants of Table I, measured baryon masses and weak hamiltonian densities as ocurring in eqs. (5) and (6) above.

Table II: Observed hyperon decay amplitudes

		$(s-wave)10^6A$	$(p-wave)10^6B$
(Λ^o_-)	$\Lambda\to p\pi^-$	0.323 ± 0.002	2.20 ± 0.05
(Λ^o_o)	$\Lambda\to n\pi^o$	-0.237 ± 0.003	-1.59 ± 0.14
(Σ^+_o)	$\Sigma^+\to p\pi^o$	-0.326 ± 0.011	2.67 ± 0.15
(Σ^+_+)	$\Sigma^+\to n\pi^+$	0.014 ± 0.003	4.22 ± 0.01
(Σ^-_-)	$\Sigma^-\to n\pi^-$	0.427 ± 0.002	-0.14 ± 0.02
(Ξ^-_-)	$\Xi^-\to \Lambda\pi^-$	-0.450 ± 0.002	1.75 ± 0.06
(Ξ^o_o)	$\Xi^o\to \Lambda\pi^o$	0.344 ± 0.006	-1.22 ± 0.07

III. DECUPLET WEAK CORRECTIONS

Recall that the decuplet 33 $\Delta(1232)$ resonance plays a 30% role in low energy πN scattering and it was the origin of the Chew-Low model [14] of crossing, analyticity and unitarity, leading to dispersion relations in particle physics. For the four observed [1] strong decays $\Delta\to N\pi$, $\Sigma^*\to\Lambda\pi$, $\Sigma\pi$, and $\Xi^*\to\Xi\pi$, the strong interaction hamiltonian density is

$$H_{DBP}=\frac{2g_{DBP}}{(m_D+m_B)}p^B\cdot\overline{D}^{(abc)}\epsilon_{cde}B^e_bP^d_a\ , \tag{8}$$

with $g_{DBP}=g_2\approx 15.7$ from data (relative to $g_{BBP}=g_{\pi NN}=g\approx 13.4$). This is consistent with the sign $g_2g>0$ found from photoproduction experiments and from SU(6) currents. Note too that $g_2\approx 15.7$ is near Hoehler's narrow Δ width value $g^*_{\Delta N\pi}\approx 1.83\ m_\pi^{-1}\approx 13.1\ GeV^{-1}$, with here $g^*_{\Delta N\pi}=2g_2(m_\Delta+m_N)^{-1}\approx 14.5\ GeV^{-1}$. This will justify our later neglect of decuplet widths, with g_2 10% higher than Hoehler's estimate.

For the analogue weak pv, pc transition D$\to$Bπ one writes [4, 15]

$$<B\mid H_w^{pv,pc}|D>=h_{2,3}\overline{B}^b_e\epsilon^{c2e}D_{(3bc)}, \tag{9}$$

and shortly we will demonstrate in many ways that this dimensionless pv weak DB scale is [4]

$$h_2\approx -0.2\cdot 10^{-6}. \tag{10}$$

However, the similar pc weak DB scale turns out to be a factor of 2 greater [15]:

$$h_3\sim -0.4\cdot 10^{-6}. \tag{11}$$

Then using soft pion or hard pion techniques combined with dispersion relations (instead using field theory spin 3/2 poles one obtains additional nonunique parameters A and Z [16]), the resulting decuplet pole pv and pc amplitudes A_{10} and B_{10} have the form for $B_i\to B_f\pi$ [4]

$$A_{10},B_{10}=\frac{(m_{B_i}\mp m_{B_f})}{3}[g_{B_fD\pi}\frac{(m_D\pm m_{B_i})}{m_D^2}H^{pv,pc}_{DB_i}+H^{pv,pc}_{B_fD'}\frac{(m_{D'}\pm m_{B_f})}{m^2_{D'}}]g_{D'B_i\pi}. \tag{12}$$

To confirm the $<B|H_w|D>$ weak scales in (9, 10), first one extracts the h_2 scale from $\Omega\to\Xi\pi$ data, by defining the pc and pv amplitudes as

$$<\pi\Xi|H_w|\Omega>=-\overline{u}(\Xi)(E_{pc}+i\gamma_5F_{pv})p^B_\mu u^\mu(\Omega), \tag{13}$$

$$|E_{pc}|\approx(m_\Omega+m_\Xi)^{-1}[24\pi m^2_\Omega\Gamma_{\Omega\Xi\pi}/p^3]^{1/2}, \tag{14}$$

giving $|E_{pc}|\approx 1.33\cdot 10^{-6}\ GeV^{-1}$ from measured $\Omega^-\to\Xi^o\pi^-$ weak decay [1]. Then CA-PCAC requires [12, 15]

$$|E_{pc}(\Xi^o\pi^-)|\approx|h_2/\sqrt{2}f_\pi|,|h_2|\approx 0.18\cdot 10^{-6}. \tag{15}$$

This h_2 scale is supported by the K^o tadpole pictures of $|<\pi\pi|H^{pv}_w|K^o>|$ and $|<\Xi^-|H^{pc}_w|\Omega^->|=|h_2|$, yielding [12, 15, 17]

$$|h_2| \approx |2g_{\Omega\Xi K^o} < 2\pi^o|H_w|K^o > f_\pi^2/m_K^2| \approx 0.23 \cdot 10^{-6}, \tag{16}$$

for the coupling $g_{\Omega\Xi K^o} \approx \sqrt{2}/f_K \approx 12.5\ GeV^{-1}$ and observed amplitude [1] $| < 2\pi^o|H_w|K^o > | \approx 26.26 \cdot 10^{-8}\ GeV$.

Also the GIM constituent quark model using SU(6) wave functions predicts [17]

$$|h_2|_{s\to d} \approx \frac{G_F s_1 c_1}{8\pi^2 3\sqrt{3}}(\frac{m_\Omega + m_\Xi}{m_\Omega - m_\Xi})(\frac{m_s - m_d}{m_s + m_d})\frac{m_\Omega}{m_d} \approx 0.15 \cdot 10^{-6}. \tag{17}$$

Given the compatibility between the pv DB scales in (15, 16, 17) we extend the latter quark model picture to estimate the pc DB scales as [15]

$$|h_3| = |\frac{m_K^2}{m_\kappa^2} h_2 \frac{< 0|H_w^{pc}|\kappa >}{< 0|H_w^{pv}|K >}| \sim |\frac{m_s + m_d}{m_s - m_d} h_2 \frac{m_K^2}{m_\kappa^2}| \sim 0.4 \cdot 10^{-6} \tag{18}$$

for $m_s/m_d \approx 1.45$ and a kappa mass of 850-900 MeV [18].

With hindsight, always using dispersion theory and unitarity but with pseudoscalar PBB and PDB couplings, our chiral weak V-A, CA-PCAC scheme will be renormalized in tree order and NO counterterms will be needed when the imaginary parts of the weak hyperon amplitudes are evaluated on mass shell via unitarity (as required using dispersion theory). Contrast this with axial-vector divergences used for ABB and ADB couplings (needed for the ChPT scheme) which lead to an unrenormalizable theory involving many counterterms.

IV. DETERMINING THE D/F RATIOS AND $< B'|H_w^{pc}|B > h_1$ SCALE

Recall that the strong d/f ratio for axial vector couplings is known to be $(d/f)_A \approx 1.74$, near the SU(6) value 1.50 and the value used for ChPT [5-7]. However for the soft pion CA-PCAC method involving pseudoscalar PBB couplings of (6) needed for the B_8 amplitudes listed in Table I, it was stressed in refs. [4] that the strong d/f ratio of $(d/f)_P \approx 2.1$ is more appropriate.

Given the observed [1] fourteen s-wave and p-wave amplitude listed above in Table II, a good fit in Tables III follow from $A = A_{cc} + A_{10}$ and $B = B_8 + B_{10}$ (along with the small $\Lambda'(1405)$ corrections to the Σ_+^+ and Σ_-^- amplitudes) developed in refs. [4] and in Secs. 2 and 3 and listed in the appendix. This fit depends on two additional parameters $(d/f)_w$ and h_1 in the pc weak hamiltonian eq.(3) assuming the values $(d/f)_w \approx -0.811$ and $h_1 \approx 22\ eV$. Thus we must first attempt to deduce these two parameters from data *other* than from $B \to B'\pi$ weak decays so that the good fits in Tables III are actually *predictions*.

Specifically we study $\Omega^- \to \Lambda K^-$ weak decay, with present decay rate [1]

$$\Gamma(\Omega^- \to \Lambda K^-) \approx \frac{p^3}{12\pi m_\Omega}(E_\Lambda + m_\Lambda)|E(\Lambda K^-)|^2 \approx 5.43 \cdot 10^{-12} MeV, \tag{19}$$

predicting the pc amplitude [15]

$$|E(\Lambda K^-)| \approx 4.27 \cdot 10^{-6} GeV^{-1}. \tag{20}$$

Assuming E in (20) to be positive and subtracting off the cc term $E_{cc} = \sqrt{3}h_2/2f_K \approx -1.53 \cdot 10^{-6}\ GeV^{-1}$ together with the small s-channel octet pole term (eq. (17a) in [15]), the resulting u-channel octet pole amplitude has the form of the $B \to B'\pi$ amplitudes in eq.(3) with $g_{\Omega\Xi^o\kappa^-} = g_2 \approx 15.7$:

$$|E(\Lambda K^-)|_{u-chan.} = \frac{h_1}{2}\frac{3}{2}(f - \frac{1}{3}d)_w \frac{2g_{\Omega\Xi^o K^-}}{(m_{\Xi^o} - m_\Lambda)(m_\Omega + m_{\Xi^o})} \approx 5.88 \cdot 10^{-6} GeV^{-1}, \tag{21}$$

$$h_1(f - \frac{1}{3}d)_w \approx 148.4 eV. \tag{22}$$

Another constraint on h_1 and $(d/f)_w$ follows from (small) $\Xi^- \to \Sigma^-\gamma$ weak radiative decay, since the dominant $\Delta I = 1/2$ W-exchange quark graph then vanishes [19, 12, 15]:

$$< \Sigma^-|H_w^{pc}|\Xi^- >_{w-ex} = \frac{1}{2}h_1(d + f)_w = 0, (d/f)_w \to -1. \tag{23}$$

The still smaller $\Delta I = 1/2$ single quark line (SQL) s$\to$d graph corresponds [20] instead to $(d/f)_w = 0$, shifting the net hadronic d/f towards [4] $(d/f)_w \approx -0.88$ or -0.86 or lower as found from gluon corrections [21] to $(d/f)_w \sim -0.8$. The analog W-exchange $B \to B'\pi$ configuration corresponds [20] to $\Sigma_o^+ + \sqrt{3}\Lambda_-^o$, for the s-wave charge commutator amplitudes requiring the combination

$$A_{cc}(\Sigma_o^+) + \sqrt{3}A_{cc}(\Lambda_-^o) = \frac{h_1}{2f_\pi}(d+f)_w, \tag{24}$$

the same $(d+f)_w$ structure as in (23). Subtracting the decuplet s-wave combination $(0.114 \cdot 10^{-6})$ from the s-wave data combination from Table II $(0.233 \cdot 10^{-6})$, the still smaller cc combination in (24)leads to (for $f_\pi \approx 93$ MeV):

$$h_1(d+f)_w \approx 22.1eV. \tag{25}$$

Combining the constraint equations (22) and (25) we find

$$h_1 \approx 22.1eV, \ (d/f)_w \approx -0.811, \tag{26}$$

$$d_w = -4.29, \ f_w = 5.29\,. \tag{27}$$

Then we predict from (26, 27) and the $< B'|H_w^{pc}|B >= H_{B'B}$ SU(3) matrix elements of eq.(3):

$$H_{n\Lambda} = -\frac{1}{2}h_1\sqrt{\frac{3}{2}}(f + \frac{1}{3}d)_w \approx -52.2eV, \tag{28a}$$

$$H_{p\Sigma^+} = -\sqrt{2}H_{n\Sigma^o} = -\frac{1}{2}h_1(f-d)_w \approx -105.9eV, \tag{28b}$$

$$H_{\Lambda\Xi^o} = +\frac{1}{2}h_1\sqrt{\frac{3}{2}}(f - \frac{1}{3}d)_w \approx 90.9eV, \tag{28c}$$

$$H_{\Sigma^-\Xi^-} = -\sqrt{2}H_{\Sigma^o\Xi^o} = +\frac{1}{2}h_1(f+d)_w \approx 11.1eV, \tag{28d}$$

from which the A_{cc} and B_8 amplitudes listed in Appendix A and are tabulated in Tables III in the concluding section.

Finally, the Σ_+^+ and Σ_-^- decays tabulated in Table II also receive small contributions from the $\Lambda(1405) = \Lambda'$ resonance [1] with estimated amplitudes enhanced by 25% relative to Tables I, II of ref. [4] because the present observed rate [1] of $\Lambda' \to \Sigma\pi$ has increased by 25% in 1998 to 50 MeV. For s and p waves we predict

$$A_{\Lambda'}(\Sigma_+^+) = A_{\Lambda'}(\Sigma_-^-) = \frac{g_{\Lambda'\Sigma\pi} < n|H_w^{pv}|\Lambda' > (m_\Sigma - m_n)}{(m_{\Lambda'} - m_n)(m_{\Lambda'} - m_\Sigma)} \sim 0.08 \cdot 10^{-6} \tag{29}$$

$$B_{\Lambda'}(\Sigma_+^+) = B_{\Lambda'}(\Sigma_-^-) = \frac{g_{\Lambda'\Sigma\pi} < n|H_w^{pc}|\Lambda' > (m_\Sigma + m_n)}{(m_{\Lambda'} + m_n)(m_{\Lambda'} - m_\Sigma)} \sim 0.14 \cdot 10^{-6}, \tag{30}$$

where $g_{\Lambda'\Sigma\pi} \approx 0.94$ and $H_{n\Lambda}^{pv,pc} \sim 50eV$ are the extensions of ref. [4].

V. CONCLUSION

In this paper we have summarized the up-dated CA-PCAC chiral approach to nonleptonic weak $B \to B'\pi$ decays. Section 2 reviews the chiral, SU(3) and $\Delta I = 1/2$ structure assuming pseudoscalar PBB couplings in the lead terms. Section 3 reviews the ~30% decuplet corrections for both s and p wave amplitudes. Rather than search for the best fit to the seven s wave and seven p wave measured $B \to B'\pi$ amplitudes of Table II, we use additional $\Omega^- \to \Lambda K^-$, $\Xi \to \Sigma\gamma$ and $\Sigma_o^+ + \sqrt{3}\Lambda_-^o$ data to *predict* (using *no* free parameters) these hyperon decays in Tables III at the pc scale $h_1 \approx 22.1eV$ with $(d/f)_w \approx -0.811$. The Appendix A displays all of the tree-level amplitudes needed.

We contrast this simple 30 year old CA-PCAC approach (using pseudoscalar PBB couplings) with the more recent and much more complex ChPT one-loop order scheme of ref. [7] (using pseudo-vector couplings). In Table IV we display the resulting one-loop order $B \to B'\pi$ amplitudes and their associated nonanalytic corrections [7]. As noted in refs [5-7], ChPT appears not to account for both s and p wave amplitudes even though this approach contains more parameters than the number of amplitudes observed.

In fact in a more recent paper [22], the authors of ref. [7] begin by stating "In our recent paper a (ChPT) calculation was performed which included all terms at one-loop order. This work suffers from the fact, however, that at this order too many new unknown LECs (low energy constants) enter the calculation so that the theory lacks predictive power." Instead in ref. [22] these authors study ChPT in tree order, but consider only $\frac{1}{2}^-$and the Roper $\frac{1}{2}^+$resonant states, while now ignoring the (usually dominant) $\frac{3}{2}^+\Delta(1232)$ resonance.

In this paper we also have worked only at (dispersion theory) tree level but found that SU(3) decuplet states analogous to the $\Delta(1232)$ play a major role, as long expected. In Appendix B, we demonstrate that πN photoproduction data clearly shows that the $\Delta(1232)$ (and not the 1/2 resonances) is the dominant resonance, suggesting a similar pattern for nonleptonic weak decays.

With hindsight, other chiral theories based on QCD also lead to a reasonable picture of hyperon nonleptonic weak decays (except for the mismatch between s- and p-wave amplitudes [23], [24]). Yet even then the $\frac{1}{2}^+$baryon resonances play a more important role in p-waves than do $\frac{1}{2}^-$baryon resonances play in s-waves [25] (just as we shall note in Appendix B for photoproduction chiral amplitudes).

It is worth mentioning that our approach, which succesfully describes hyperon nonleptonic decays, may be usefully applied in the hypernuclear decay calculations [26].

In summary, using on-shell dispersion theory-unitarity techniques, the tree-level $B \to B'\pi$ weak hyperon amplitudes listed in Apendix A involve no counterterms (as opposed to ChPT). Moreover such graphs a-priori obey (weak interaction) chiral symmetry since the current-comutator amplitudes of (5) manifestly obey the chiral relations (1) and also the octet (decuplet) pole terms satisfy the chiral relations (6) and (12), respectively. It is satisfying that this simple chiral model (involving no free parameters) predicts 14 (s- and p-wave) amplitudes which reasonably match data [1] as listed in Tables IIIab.

Table III a: B→B′π s wave CA-PCAC predictions 10^6A

	$\underline{cc}$	$\underline{10}$	$\underline{\Lambda'}$	$\underline{Theory}$	$\underline{Data}$
Λ^o_-	0.397	−0.092		0.305	0.323 ± 0.002
Λ^o_o	−0.281	0.065		−0.216	-0.237 ± 0.003
Σ^+_o	−0.569	0.273		−0.296	-0.326 ± 0.011
Σ^+_+	0	−0.086	0.08	~ 0	0.014 ± 0.003
Σ^-_-	0.805	−0.486	0.08	~ 0.40	0.427 ± 0.002
Ξ^-_-	−0.691	0.223		−0.468	-0.451 ± 0.002
Ξ^o_o	0.489	−0.153		0.336	0.344 ± 0.006

Table III b: B→B′π p wave CA-PCAC predictions 10^6 B

	$\underline{8}$	$\underline{10}$	$\underline{\Lambda'}$	$\underline{Theory}$	$\underline{Data}$
Λ^o_-	2.17	0.41		2.58	2.20 ± 0.05
Λ^o_o	−1.56	−0.29		−1.85	-1.59 ± 0.14
Σ^+_o	3.16	−0.39		2.77	2.67 ± 0.15
Σ^+_+	3.94	−0.13	0.14	~ 3.95	4.22 ± 0.01
Σ^-_-	−0.58	0.28	0.14	~ -0.16	-0.14 ± 0.02
Ξ^-_-	1.89	−0.44		1.45	1.75 ± 0.06
Ξ^o_o	−1.31	0.32		−0.99	-1.22 ± 0.07

Table IV: ChPT one-loop order B→B′π amplitudes [7]

	Theory	*Data*
$A(\Lambda^o_-)$	$0.333+0.208$	0.323 ± 0.002
$A(\Sigma^+_+)$	$0+0$	0.014 ± 0.003
$A(\Sigma^-_-)$	$0.437+0.129$	0.427 ± 0.002
$A(\Xi^-_-)$	$-0.434-0.174$	-0.450 ± 0.002
$B(\Lambda^o_-)$	$-2.59-0.30$	2.20 ± 0.05
$B(\Sigma^+_+)$	$0.15+9.61$	4.22 ± 0.01
$B(\Sigma^-_-)$	$0.74-0.35$	-0.14 ± 0.02
$B(\Xi^-_-)$	$0.74+1.02$	1.75 ± 0.06

APPENDIX A: SU(3) STRUCTURE OF A_{CC}, A_{10}, B_8, B_{10}.

We display all s wave and p wave amplitudes A, B contributing to eqs. (7) with masses m_p, m_n denoted by p, n, etc. and $f_\pi \approx 93$ MeV. The predicted weak hamiltonian matrix elements in eqs. (28) and strong couplings in Table I are always used. Both A and B are weighted by 10^{-6}.

$$A_{cc}(\Lambda^o_-) = -\sqrt{2}A_{cc}(\Lambda^o_o) = -\frac{1}{\sqrt{2}f_\pi}H_{n\Lambda} \approx 0.397,$$

$$A_{cc}(\Sigma^+_o) = \frac{1}{2f_\pi}H_{p\Sigma^+} \approx -0.569,$$

$$A_{cc}(\Sigma^+_+) = -\frac{1}{f_\pi}(H_{n\Sigma^o} + \frac{1}{\sqrt{2}}H_{p\Sigma^+}) = 0,$$

$$A_{cc}(\Sigma^-_-) = \frac{1}{f_\pi}H_{n\Sigma^o} \approx 0.805,$$

$$A_{cc}(\Xi^-_-) = -\sqrt{2}A_{cc}(\Xi^o_o) = -\frac{1}{\sqrt{2}f_\pi}H_{\Lambda\Xi^o} \approx -0.691.$$

$$A_{10}(\Lambda^o_-) = -\sqrt{2}A_{10}(\Lambda^o_o) = \frac{h_2 g_2}{3\sqrt{6}}(\Lambda - p)\frac{(\Sigma^{*+}+p)}{(\Sigma^{*+})^2} \approx -0.092$$

$$A_{10}(\Sigma^+_o) = \frac{-\sqrt{2}h_2 g_2}{9}(\Sigma^+ - p)[\frac{\Delta^+ + \Sigma^+}{(\Delta^+)^2} + \frac{1}{2}\frac{\Sigma^{*+}+p}{(\Sigma^{*+})^2}] \approx 0.273$$

$$A_{10}(\Sigma^+_+) = \frac{h_2 g_2}{9}(\Sigma^+ - n)[\frac{\Delta^+ + \Sigma^+}{(\Delta^+)^2} - \frac{1}{2}\frac{\Sigma^{*o}+n}{(\Sigma^{*o})^2}] \approx -0.086$$

$$A_{10}(\Sigma^-_-) = \frac{h_2 g_2}{3}(\Sigma^- - n)[\frac{\Delta^- + \Sigma^-}{(\Delta^-)^2} + \frac{1}{6}\frac{\Sigma^{*o}+n}{(\Sigma^{*o})^2}] \approx -0.486$$

$$A_{10}(\Xi^-_-) = -\sqrt{2}A_{10}(\Sigma^o_o) = -\frac{h_2 g_2}{3\sqrt{6}}(\Xi^{-,o} - \Lambda)[\frac{\Sigma^{*-,o}+\Xi^{-,o}}{(\Sigma^{*-,o})^2} + \frac{\Xi^{*o}+\Lambda}{(\Xi^{*o})^2}] \approx 0.223, -0.216$$

$$B_8(\Lambda^o_-) = -(\Lambda + p)[\frac{g_{\pi^- pn}H_{n\Lambda}}{(p+n)(\Lambda - n)} - \frac{g_{\pi^-\Sigma^+\Lambda}H_{p\Sigma^+}}{(\Lambda+\Sigma^+)(\Sigma^+ - p)}] \approx 2.17$$

$$B_8(\Lambda^o_o) = -(\Lambda + n)[\frac{g_{\pi^o nn}H_{n\Lambda}}{2n(\Lambda - n)} - \frac{g_{\pi^o\Sigma^o\Lambda}H_{n\Sigma^o}}{(\Lambda + \Sigma^o)(\Sigma^o - n)}] \approx -1.56$$

$$B_8(\Sigma^+_o) = -(\Sigma^+ + p)[\frac{g_{\pi^o pp}H_{p\Sigma^+}}{2p(\Sigma^+ - p)} - \frac{g_{\pi^o\Sigma^+\Sigma^+}H_{p\Sigma^+}}{2\Sigma^+(\Sigma^+ - p)}] \approx 3.16$$

$$B_8(\Sigma^+_+) = -(\Sigma^+ + n)[\frac{g_{\pi^+ pn}H_{p\Sigma^+}}{(p + n)(\Sigma^+ - p)} - \frac{g_{\pi^+\Sigma^+\Sigma^o}H_{n\Sigma^o}}{(\Sigma^+_+ + \Sigma^o)(\Sigma^o - n)} - \frac{g_{\pi^+\Sigma^+\Lambda}H_{n\Lambda}}{(\Lambda + \Sigma^+)(\Lambda - n)}] \approx 3.94$$

$$B_8(\Sigma^-_-) = (\Sigma^- + n)[\frac{g_{\pi^-\Sigma^-\Sigma^o}H_{n\Sigma^o}}{(\Sigma^o + \Sigma^-)(\Sigma^o - n)} + \frac{g_{\pi^-\Lambda\Sigma^-}H_{n\Lambda}}{(\Lambda + \Sigma^-)(\Lambda - n)}] \approx -0.58$$

$$B_8(\Xi^-_-) = -(\Xi^- + \Lambda)[\frac{g_{\pi^-\Sigma^-\Lambda}H_{\Sigma^-\Xi^-}}{(\Lambda + \Sigma^-)(\Xi^- - \Sigma^-)} - \frac{g_{\pi^-\Xi^-\Xi^o}H_{\Lambda\Xi^o}}{(\Xi^- + \Xi^o)(\Xi^o - \Lambda)}] \approx 1.89$$

$$B_8(\Xi^o_o) = -(\Xi^o + \Lambda)[\frac{g_{\pi^o\Sigma^o\Lambda}H_{\Sigma^o\Xi^o}}{(\Lambda + \Sigma^o)(\Xi^o - \Sigma^-)} - \frac{g_{\pi^o\Xi^o\Xi^o}H_{\Lambda\Xi^o}}{2\Xi^o(\Xi^o - \Lambda)}] \approx -1.31$$

$$B_{10}(\Lambda^o_-) = -\sqrt{2}B_{10}(\Lambda^o_o) = \frac{-h_3 g_2}{3\sqrt{6}}(\Lambda + p)[\frac{\Sigma^{*+} - p}{(\Sigma^{*+})^2}] \approx 0.408$$

$$B_{10}(\Sigma^+_o) = \frac{\sqrt{2}h_3 g_2}{9}(\Sigma^+ + p)[\frac{\Delta^+ - \Sigma^+}{(\Delta^+)^2} + \frac{1}{2}\frac{\Sigma^{*+} - p}{(\Sigma^{*+})^2}] \approx -0.384$$

$$B_{10}(\Sigma^+_+) = -\frac{h_3 g_2}{9}(\Sigma^+ + n)[\frac{\Delta^+ - \Sigma^+}{(\Delta^+)^2} - \frac{1}{2}\frac{\Sigma^{*o} - n}{(\Sigma^{*o})^2}] \approx -0.132$$

$$B_{10}(\Sigma^-_-) = -\frac{h_3 g_2}{3}(\Sigma^- + n)[\frac{\Delta^- - \Sigma^-}{(\Delta^-)^2} + \frac{1}{6}\frac{\Sigma^{*o} - n}{(\Sigma^{*o})^2}] \approx 0.282$$

$$B_{10}(\Xi^-_-) = -\sqrt{2}B_{10}(\Xi^o_o) = \frac{h_3 g_2}{3\sqrt{6}}(\Xi^{-,o} + \Lambda)[\frac{\Sigma^{*-,o} - \Xi^{-,o}}{(\Sigma^{*-,o})^2} + \frac{\Xi^{*-,o} - \Lambda}{(\Xi^{*-,o})^2}] \approx -0.442, -0.447$$

APPENDIX B: PION PHOTOPRODUCTION SOFT PION CHIRAL THEOREM AND THE DOMINANCE OF THE Δ ISOBAR.

First we state the FFR soft-pion theorem [27]

$$\overline{A_1}^{(+)}(\nu = t = q^2 = 0) = -\frac{g_{\pi NN}\kappa^v}{4\pi m_N^2} \approx -0.27 m_\pi^{-2}\,, \tag{B.1}$$

where $A_1^{(+)}$ is the first isotopic-even pion photoproduction background amplitude (weighted by $\overline{u}_{N'}\ \frac{1}{2}[\gamma.k,\, \gamma_\nu]\,\gamma_5 u_N$). Next we saturate this background via the $\frac{3}{2}^+\Delta(1232)$, $\frac{1}{2}^+N(1440)$, $\frac{1}{2}^-N(1520)$ and $\frac{1}{2}^-N(1535)$ resonant states [28]:

$$\overline{A}_{1,\Delta}^{(+)}(q \to 0) = -\frac{g^*_\Delta(m_\Delta + m_N)}{6 m_\Delta m_N}(G^*_M - 3G^*_E) \approx -0.28\, m_\pi^{-2}$$

$$\overline{A}^{(+)}_{1,\,1440}\,(q\to 0) = -\frac{g'_{\kappa'_v}}{2m_{N^*}(m_{N^*}+m_N)} \approx 0.03\,m_\pi^{-2}$$

$$\overline{A}^{(+)}_{1,\,1520}\,(q\to 0) = -\frac{g^*_N(m_{N^*}-m_N)}{16m_{N^*}m_N}(G'_{E^v}-3G'_{M^v}) \approx -0.01\,m_\pi^{-2}$$

$$\overline{A}^{(+)}_{1,\,1535}\,(q\to 0) = -\frac{g''_{\kappa''_v}}{2m_{N^*}(m_{N^*}-m_N)} \approx -0.01\,m_\pi^{-2} \tag{B.2}$$

with $g^*_\Delta = g^*_{\pi N\Delta} \approx 2.12\,m_\pi^{-1}$ and

$$G^*_M = -\frac{1}{e}\left[\frac{2}{3}\left(\frac{m_N^3}{m_\Delta^2-m_N^2}\right)\right]^{\frac{1}{2}}(3A_{\frac{3}{2}}+\sqrt{3}A_{\frac{1}{2}}) \approx 3.07$$

$$G^*_E = -\frac{1}{e}\left[\frac{2}{3}\left(\frac{m_N^3}{m_\Delta^2-m_N^2}\right)\right]^{\frac{1}{2}}(A_{\frac{3}{2}}-\sqrt{3}A_{\frac{1}{2}}) \approx 0.05, \tag{B.3}$$

where the latter helicity couplings $A_{\frac{1}{2},\frac{3}{2}}$ are listed in the 1980 PDG tables [29]. The much smaller couplings for the N(1440), N(1520) and N(1535) are derived or listed in refs. [28].

Note that the $\Delta(1232)$ isobar generates the dominant photoproduction background resonance term in (B.2), yet the minor N resonant contributions still help to sum the four terms in (B.2) to be the precise match of the FFR chiral result (B.1). This suggests that the analogue SU(3) decuplet resonant terms together with the SU(3) octet and current algebra (chiral) terms tabulated in Appendix A above should (and do) match all 14 measured hyperon nonleptonic weak decay s- and p-wave amplitudes.

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